CSE 390B, 2024 Winter

Building Academic Success Through Bottom-Up Computing

# Time Management & Boolean Arithmetic

Time Management, Overview of Numbers in Binary, Boolean Arithmetic, Circuits for Adding Binary Numbers

# Project 2 Check-in

- How has Project 2 been coming along?
- What questions do you have about Project 2?
- Remember to double check your submission on GitLab
  - Navigate to GitLab, open tags, and verify that the associated commit includes your expected changes

# Lecture Outline

- **Time Management**
  - **Identifying Weekly Time Commitments**
- Overview of Numbers in Binary
  - Comparison Between Binary and Decimal
- Boolean Arithmetic
  - Addition Operator and Handling Binary Overflow
- Circuits for Adding Binary Numbers
  - Overview of the Half Adder and Full Adder

# Time Management

- One of your most valuable resources in college is time

- What typically fills up your time during the quarter?
  - Lectures, quiz sections, and attending office hours
  - Part-time jobs and working
  - Studying
  - Extracurricular activities or RSOs
  - Commuting
  - Chores at home
  - Socializing with friends and family
  - Physical, mental, spiritual activities

# Weekly Time Commitments

- Class meeting times and quiz sections
- Family, friends, community, extracurricular commitments
- Physical, mental, social, spiritual activities
- Studying for each of your classes
  - The number of credits for a course reflects the number of hours the class meets
  - In general, courses require two hours of homework for every one hour of class
- What else is not reflected given your specific situation?

# Tracking Weekly Time Commitments

- We often don't realize what takes up our time until we manually track how we use our time
- Exercise: Complete the weekly time commitments table
- Tip: Use different colors for different activity types

Weekly Time Commitments Table Name: _____

| Time | Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|---|
| 7:30am | | | | | | | |
| 8:00am | | | | | | | |
| 8:30am | | | | | | | |
| 9:00am | | | | | | | |
| 9:30am | | | | | | | |
| 10:00am | | | | | | | |
| 10:30am | | | | | | | |
| 11:00am | | | | | | | |
| 11:30am | | | | | | | |
| 12:00pm | | | | | | | |
| 12:30pm | | | | | | | |
| 1:00pm | | | | | | | |
| 1:30pm | | | | | | | |
| 2:00pm | | | | | | | |
| 2:30pm | | | | | | | |
| 3:00pm | | | | | | | |
| 3:30pm | | | | | | | |
| 4:00pm | | | | | | | |
| 4:30pm | | | | | | | |
| 5:00pm | | | | | | | |
| 5:30pm | | | | | | | |
| 6:00pm | | | | | | | |
| 6:30pm | | | | | | | |
| 7:00pm | | | | | | | |
| 7:30pm | | | | | | | |
| 8:00pm | | | | | | | |
| 8:30pm | | | | | | | |
| 9:00pm | | | | | | | |
| 9:30pm | | | | | | | |
| 10:00pm | | | | | | | |
| 10:30pm | | | | | | | |
| 11:00pm | | | | | | | |
| 11:30pm | | | | | | | |

# Time Management Group Discussion

Now that you've filled out your weekly time commitments, discuss the following in groups for 4-6 minutes:

- Does anything surprise you about the way you spend your time?
- What might you change about the way you utilize your time?
- How could you use this time commitments sheet in the future?

# Lecture Outline

- Time Management
  - Identifying Weekly Time Commitments
- **Overview of Numbers in Binary**
  - **Comparison Between Binary and Decimal**
- Boolean Arithmetic
  - Addition Operator and Handling Binary Overflow
- Circuits for Adding Binary Numbers
  - Overview of the Half Adder and Full Adder

# What is Binary?

- A **base n** number system is a system of number representation with n **symbols**

- Decimal system is a **base 10** number system
  - Base 10 symbols: $0$, $1$, $2$, $3$, $4$, $5$, $6$, $7$, $8$, $9$ (each called a **digit**)
  - Increase a number by moving to the next greatest symbol
  - Add another digit when we run out of symbols

- Binary is a **base 2** number system
  - Base 2 symbols: $0$, $1$ (each called a **bit**)
  - Often prefixed with $0\text{b}$ (e.g., $0\text{b}1101$, $0\text{b}10$)
  - **Least-significant bit (LSB):** Lowest-order position of a binary value
  - **Most-significant bit (MSB):** Highest-order position of a binary value

# Representing Numbers in Base 2

- Binary numbers are identical, except in base 2
  - Describe a value by specifying multiples of powers of 2
  - For example, a breakdown of $0b1101$ in binary (13 in decimal)

| Binary | Power of 2 |
|---|---|
| $0b1000$ | $1 \times 2^3$ |
| $0b0100$ | $1 \times 2^2$ |
| $0b0000$ | $0 \times 2^1$ |
| $0b0001$ | $1 \times 2^0$ |

# Binary vs. Decimal

| Binary | Decimal |
|---|---|
| 0b000 | 0 |
| 0b001 | 1 |
| 0b010 | 2 |
| 0b011 | 3 |
| 0b100 | 4 |
| 0b101 | 5 |
| 0b110 | 6 |
| 0b111 | 7 |
| … | … |

Lecture 3: Time Management & Boolean Arithmetic

When poll is active respond at **PollEv.com/cse390b**

## What is the binary representation of the decimal value of 29?

0

# Lecture Outline

- Time Management
  - Identifying Weekly Time Commitments
- Overview of Numbers in Binary
  - Comparison Between Binary and Decimal
- **Boolean Arithmetic**
  - **Addition Operator and Handling Binary Overflow**
- Circuits for Adding Binary Numbers
  - Overview of the Half Adder and Full Adder

# Roadmap: Boolean Arithmetic

- Addition — Implement
- Subtraction — Get it for free!
- Comparison (<, >, ==, !=) — Get it for free!
- Multiplication — Postpone to software
- Division — Postpone to software

# Binary Addition

- How do we add two binary numbers?
  - As humans, we could convert to decimal and then back

- Example: $0\text{b}101 + 0\text{b}010$
  - First convert $0\text{b}101$ to decimal (result is 5)
  - Next convert $0\text{b}010$ to decimal (result is 2)
  - Add the decimal numbers and convert back to binary
    - $5 + 2 = 7$, which is $0\text{b}111$ in binary

- What's more useful is understanding the rules of binary addition so we can teach them to a computer

# Case Study: Decimal Addition

- Consider how we perform decimal addition
  - Right to left (least significant place to most significant place)
  - When a column's result is more than one digit, carry over the digit that overflows

- Example:

| carry | | | | |
|---|---|---|---|---|
| a | 5 | 7 | 8 | 3 |
| b | 2 | 4 | 5 | 6 |
| sum | | | | |

# Binary Addition

- Binary addition conceptually the same as decimal addition
  - Right to left (least significant place to most significant place)
  - When a column's result is more than one digit, carry over the bit that overflows

- Example:

| carry | | | | |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 1 |
| b | 0 | 1 | 0 | 1 |
| sum | | | | |

# Binary Overflow

- What if there's a carry bit in the last column?

- Example:

| carry | | | | |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 0 |
| b | 1 | 0 | 1 | 0 |
| sum | | | | |

# Binary Overflow

- What if there's a carry bit in the last column?

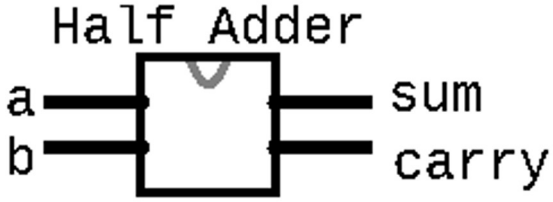
- We can't represent it in our fixed-width numbers
  - We are going to "drop" or ignore the extra carry bit

| carry | | | | | |
|---|---|---|---|---|---|
| a | | 0 | 1 | 1 | 0 |
| b | | 1 | 0 | 1 | 0 |
| sum | | | | | |

# Lecture Outline

- Time Management
  - Identifying Weekly Time Commitments
- Overview of Numbers in Binary
  - Comparison Between Binary and Decimal
- Boolean Arithmetic
  - Addition Operator and Handling Binary Overflow
- **Circuits for Adding Binary Numbers**
  - **Overview of the Half Adder and Full Adder**

# Half Adder




- Circuit for adding two bits together
- Takes in two inputs: `a`, `b`
  - `a` is the first bit being added
  - `b` is the corresponding bit to be added
- Produces two outputs: `sum`, `carry`
  - `sum` is the value to be put for this column in the result
  - `carry` is the value to be carried over to the next column

| carry | | | | |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 0 |
| b | 1 | 0 | 1 | 0 |
| sum | | | | |

```
/**
 * Computes the sum of 2 bits
 */

CHIP HalfAdder {
    IN a, b;
    OUT sum, carry;

    PARTS:
    // Put your code here:

}
```

# Half Adder Example

- Example: $0b0111 + 0b0101$

- For the right-most (least significant) column:
  - $a = 1$
  - $b = 1$
  - sum =
  - carry =

| carry | | | | |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 1 |
| b | 0 | 1 | 0 | 1 |
| sum | | | | |

# Half Adder Example

- Boolean expressions:
  - sum =
  - carry =

| a | b | sum | carry |
|---|---|---|---|
| 0 | 0 | | |
| 0 | 1 | | |
| 1 | 0 | | |
| 1 | 1 | | |

# Half Adder Group Work

- Determine the half adder logical Boolean expression
  - First, fill in the truth table values for sum and carry based on the inputs
  - Then, develop a Boolean expression for sum and carry based on the truth table

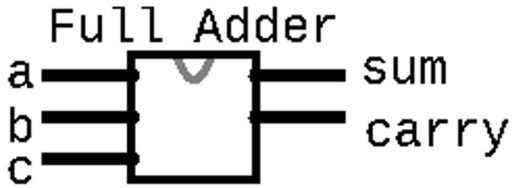
- Five-minute group discussion, identify one person to share each of the following as a large group:
  - Overview of what the half adder does
  - Thought process for reaching the Boolean expression for sum
  - Thought process for reaching the Boolean expression for carry

# Half Adder Example

- Boolean expressions:
  - sum = a XOR b
  - carry = a AND b

| a | b | sum | carry |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

# Full Adder




- Circuit for adding three bits together (two bits *and* carry bit together from previous column)
  - `a` is the first bit being added
  - `b` is the corresponding bit to be added
  - `c` is the carry bit from the right column

| carry | | | | |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 0 |
| b | 1 | 0 | 1 | 0 |
| sum | | | | |

- Produces two outputs: `sum`, `carry`
  - `sum` is the value to be put for this column in the result
  - `carry` is the value to be carried over to the next column

```
/**
 * Computes the sum of 3 bits
 */

CHIP FullAdder {
    IN a, b, c;
    OUT sum, carry;

    PARTS:
    // Put your code here:

}
```

# Full Adder

- Example: $0b0111 + 0b0101$

- For the second (second least significant) column:
  - $a = 1$
  - $b = 0$
  - $c = 1$
  - sum =
  - carry =

| | | | | |
|---|---|---|---|---|
| carry | | | 1 | |
| a | 0 | 1 | 1 | 1 |
| b | 0 | 1 | 0 | 1 |
| sum | | | | 0 |

# Full Adder Truth Table

| a | b | c | sum | carry |
|---|---|---|---|---|
| 0 | 0 | 0 | | |
| 0 | 0 | 1 | | |
| 0 | 1 | 0 | | |
| 0 | 1 | 1 | | |
| 1 | 0 | 0 | | |
| 1 | 0 | 1 | | |
| 1 | 1 | 0 | | |
| 1 | 1 | 1 | | |

Lecture 3: Time Management & Boolean Arithmetic

When poll is active respond at **PollEv.com/cse390b**

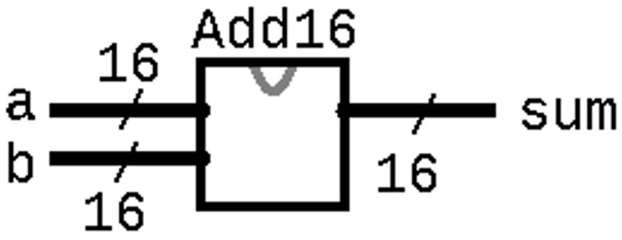
**What are the sum and carry bits when a=0, b=1, and c=1?**

0

# Full Adder Truth Table

| a | b | c | sum | carry |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Multi-Bit Adder




- Adds two 16-bit numbers

- Connects the full adders for each column together (wires the out carry from one column to the in carry of the next)

```
/**
 * Adds two 16-bit Two's Complement
 * values. Overflow is ignored.
 */
CHIP Add16 {
  IN a[16], b[16];
  OUT sum[16];

  PARTS:
  // Put your code here:

}
```

```
    … 0 1 1 1 0 0
    … 0 0 1 0 1 0 1
+   … 1 0 1 1 1 0 0
-------------------
    … 1 1 1 0 0 0 1
```

# Lecture 3 Reminders

- **Project 2 due Friday (1/12) at 11:59pm**

- Course Staff Support
  - Amy has office hours tomorrow in CSE2 151 at 1:30pm
  - Feel free to post your questions on the Ed discussion board too